A Counterintuitive Probability Problem

From Simulation to Theory

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Outline

1. Introduce the Problem
2. Describe the Activity
3. Do the Activity
4. General Principles
5. Other Examples
6. Conclusion

The Introduction

The Problem

If I start flipping a coin, which sequence of flips is more likely to occur first: HHT or HTT?

Why is this Interesting?

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- It's accessible.
- It's not intuitive.
- It's an easy way to introduce simulation as a way to build intuition, and then theory.

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- Can we physically test this?
- How do we build up our confidence that what we're seeing is actually happening?
- Simulate it.
- Ask why.

Let’s Try It!

The Problem

If I start flipping a coin, which sequence of flips is more likely to occur first: HHT or HTT?

Trials

HHT	HTT

HTT

THTHTHHT

HHT

HHT

HHT

HTHTT

HTT

TTTTHHHHT

HHHT

HHT

HHHHHHT

HTHTT

HTT

TTHTT

TTHHHT

HTHTHHT

HHHT

The Algorithm

1. Flip the coin.
2. Record the flip result.
3. Look at the most current three flips.
4. Compare the three flips with HHT and HTT.
    - If we don't get a match, go to step 1.
5. Add a tally to the "winning" pattern, clear the list of flips.

The Algorithm

Python Notebook

Results

The probability of HHT showing up first is: 66.67%
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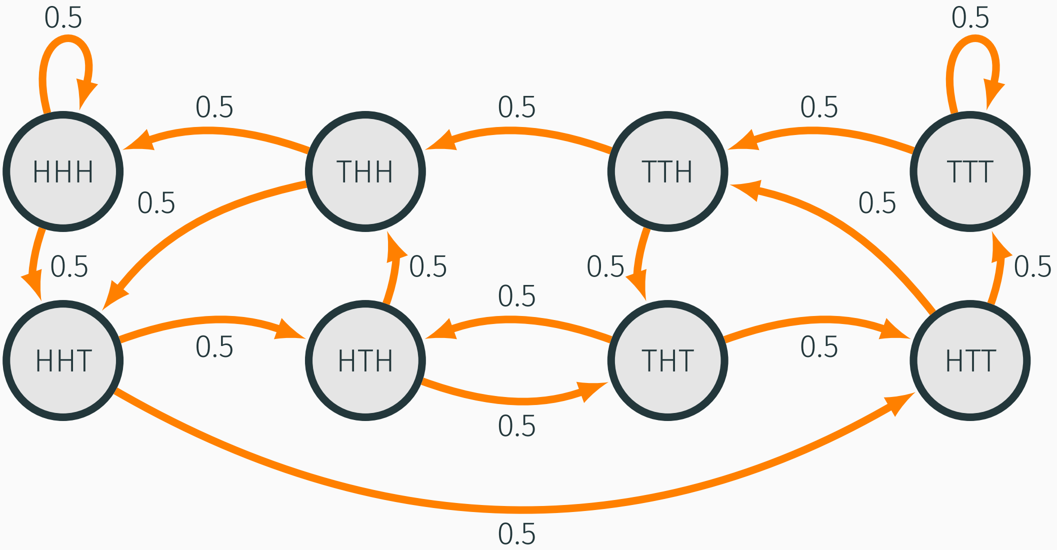
Why?

Markov Chain



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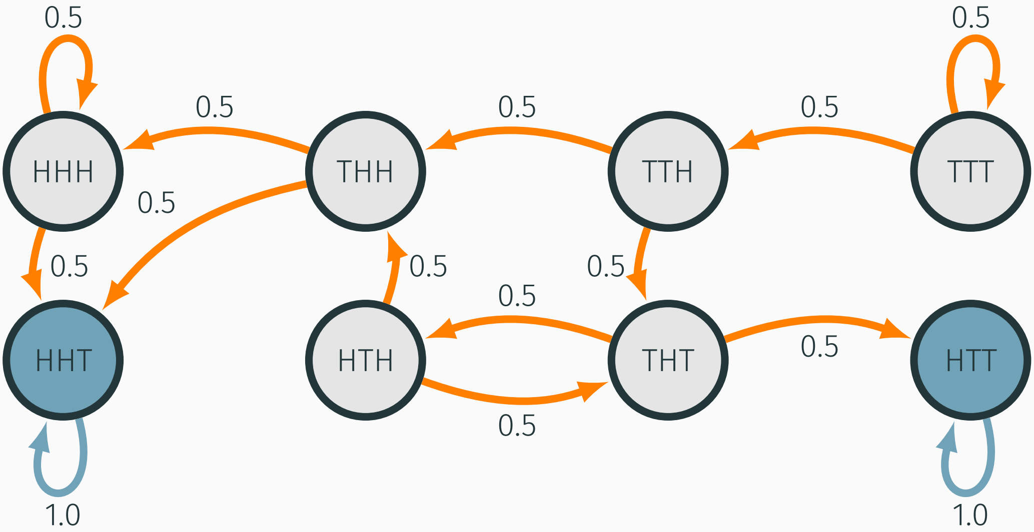

The General Ideas

From Simulation to Theory

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- The general idea can be applied in a lot of different content areas.
- "Simulation" doesn't have to be just larger sample sizes.
- Works well when we're working with intuition: challenging misplaced intuition or using student intuition to discover something.
- Easy to implement as a transition to active learning or inquiry-based learning.

Where Else Have We Used This?

Other Examples

Statistics: Central Limit Theorem

- Randomly sample from a large population of real data and find the sample mean.
- Plot the sampling distribution to compare with the population data.
- Approximate the Normal distribution.

Statistics: Hypothesis Testing

Calculus: Limits

Calculus: Definite Integrals

Finite: Simplex Method

Other Examples

Statistics: Central Limit Theorem

Statistics: Hypothesis Testing

- Randomly sample from the null distribution and compare with the observation.
- Plot all of the random sample statistics to get a distribution.
- Lead to an explanation of p-values.

Calculus: Limits

Calculus: Definite Integrals

Finite: Simplex Method

Other Examples

Statistics: Central Limit Theorem

Statistics: Hypothesis Testing

Calculus: Limits

- Numerically approximate limits with a discussion on what “arbitrarily close” and “sufficiently close” mean.
- Add precision to our approximations to whatever level we’d like.
- Introduce the $\epsilon - \delta$ definition of a limit.

Calculus: Definite Integrals

Finite: Simplex Method

Other Examples

Statistics: Central Limit Theorem

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Calculus: Limits

Calculus: Definite Integrals

- Construct Riemann Sums to approximate the areas bounded by curves.
- Evaluate Riemann Sums with very fine partition.
- Lead to a definition of the definite integral.

Finite: Simplex Method

Other Examples

Statistics: Central Limit Theorem

Statistics: Hypothesis Testing

Calculus: Limits

Calculus: Definite Integrals

Finite: Simplex Method

- Set up a feasible region for a 2D linear programming exercise, and evaluate the objective function for several points.
- Discuss how to determine if we have the "best" point. Why?
- The optimal solution occurs at corner points, extend to multi-dimension.

Thanks for Watching!

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