**Existence, Sign, Size, and Symmetry Analysis of y-values**

**in a function (or *E.S’s.A.y* for short)**

Below are a standard set of steps all student should work to habitually put before any algebraic expression they are asked to work with in order to ground and guide the symbol manipulation in a “birds eye view” big-picture graphic sensibility that will help them see errors in real time and gain a wider abstracted view for future problems. Further: Calculus student should do *ESSSAYs* not only at the f(x) level but also at the f’(x) and f’’(x) levels, marrying what they find back to its interpretation in the original f(x) function.

**E is for existence:** Know where your expression is undefined. Remember we step outside the realm of real numbers when we

divide by zero,

take the even indexed root of a negative number,

take the log of either a zero of a neg number,

Trig: divide by a zero of cos() or sin()

**S is for sign:** Where an expression is not undefined, it exists and so must be either: zero, positive, or negative. Remember to find the zeros of a function we must set y=0 and solve for x. Between undefine points and zero points continuous expressions must be either always positive or always negative.

Most of intermediate algebra is about practicing the large variety for solving methods and matching them to the types of expressions they work on. Of course the different types can then mixed by combination (+,-,\*,/) or composition(o) sometimes forcing us into numeric methods: Int. val. theorem, graphing, Newton’s method

Types and methods as loosely associated:

Polynomial/Rational ---- factoring/ synthetic and long div, LCM

Exponential/Logarithmic --- exp rules, log rules, inverse operations

Radical --- powers, complete the square

Trigonometric --- inverse trig, periodic symmetry

**S is for Size:** expressions often have different behaviors for magnitude bigger than 1 versus smaller than 1 especially in reciprocals.

3 vs 1/3 0.001 vs 1000 -1,000,000 vs -1E-6

At the limiting case we have the idea of the reciprocity of $0^{+} and \infty $ and of $0^{-} and -\infty $

 Ex. Consider the graphs of $y=\sin(x) vs y=\csc(x)$

**S is for symmetry:** While it might be best to recognize symmetry earlier in the process this is not always that easy for students; still better late than never! Symmetry, if noticed, is always a timesaver -- sometimes all but eliminating huge chucks of work.

y-axis (Even) $f(-x)=f(x)$

origin (Odd) $f(-x)=-f(x)$

periodic $f(x+P)=f(x)$

**Ay is for analysis of y-values:** The cumulative synthesis of all these critical values, intervals and relations together with the notion of limiting behavior including End Behavior gives us a wholistic view of the problem and can’t help but lend clarity to what we are seeking in the task at hand. In practical execution, a full *ESsAy* is not desirable on every problem, principally do the time-benefit considerations (*ESsAy* taking much more time and effort then the problem we’re using it for), but to not learn the method at all or to not use it in part on a regular basis would be grave mistake in that it allows for ignorant/persistent misuse of methods and thoughtlessly wrong mistakes that could be easily caught. Thus, formal and rigorous complete practice of *ESsAy* for a time is the road to intuitive mathematical understanding through developed metacognitive skills.

Examples on library function relationships