Hi, I'm Laura Watkins and I teach it at Glendale Community College in Glendale Arizona. A member of the Maricopa community colleges. I've been teaching at GCC since 2002 and I typically teach courses ranging from Multivariable calculus, Calculus, Differential Equations, Linear Algebra, and the pre-service elementary teacher courses.

I was first given the opportunity to teach the pre-service elementary teacher courses when I was a graduate student. During this experience it became apparent to me the importance and subtleties of teaching Elementary School mathematics. Given that the students that I teach will be future teachers it's important that I give them the opportunity to engage with the mathematics and to own their own knowledge of Elementary School mathematics. So I'm pleased to be here and share with you a question that I often ask myself when I go about teaching this class.

The question I often ask myself is how can I promote student ownership and engagement during the study of positional numeration system? This can typically be a dry subject for students so anything I can do to further engage the students and help them to build their own ownership of this knowledge is going to benefit them as well as their own students in the future.

I typically disgusting should numeration systems that are non positional as well as positional to provide a bit of history and context. Um, we do identify key characteristics of a positional numeration system. Um, Starting with that it consists of a base b which is a natural number greater than 1. And a set of digits or symbols that we can use to represent numbers in that base. And those digits or symbols represent the numbers between 0 and b -1. We also note that the position of a digit determines the value assigned to that digit.

And lest the students think that the study of other bases is not relevant I do remind them of some applications. Particularly the binary system which typically they're already aware of with respect to computers but also the octal numeration system which is used by some indigenous peoples and is used in aircraft transponders. And then there's the hexadecimal system which is often used in computers to identify colors.

One of the challenges of teaching structure of positional numeration systems is typically our students have been working with the decimal system for more than 10 years and they don't typically think of a written number in terms of its structure and that's an important foundation for mathematics beyond arithmetic.

So in order to help students focus on the structure of a positional numeration system we do discuss what digits are used in that system? What are the place values? And What is the relationship between neighboring place values?

So when I think about how I can promote engagement and ownership of my students with respect to numeration systems I believe this starts with something as simple as Counting. And so in my class we start with the idea of counting and counting in our base ten system.

So I often start with what are the digits that we use in our base ten system that allow us to write our numeral er numbers and so I get the standard digits that we all know and that's well and good and they all agree that these are the symbols that we use to write all of our numbers. And so then, um, on their tables they will have a collection of um these linking cubes which are very versatile and can be used to explore a number of concepts.

And we start counting these cubes, so we start with 1, 2, 3, and we keep going until we get to 9. There we have 9 cubes and then, um, we add on more, we put one more in, we count and we say 10. And so the students will be writing 10 in their on their paper. And I will say so what do I write? what am I supposed to write here? And they'll say ten and I said No. Literally, what am I supposed to write? So eventually they will tell me that I need to write a 1 followed by a zero. And then I have them turn to a partner and talk about why we write that ten that way.

Notice it's a combination of the digits we're allowed to use but why do we write it that way. I usually give them about 15 to 20 seconds to discuss this and then report out. And what I get is that we write this ten this quantity ten this way to represent that there is one group of ten and zero units.

Now, uh, these linking cubes are on the table and so often a student or more than one student they like to play with these that will go ahead and will put them together in a long rod like this. Which really nicely illustrates that this is 10, this is a group of 10.

Typically in these classes we call this when we make a rod like this we call it a Long. And so this represents one Long.

And of course we keep on counting 11, 12, 13, and so forth we'll get to 19. And eventually 20 and by this time they kind of know what I'm looking for and so they'll tell me yeah that's two groups of 10 and units or 2 of these longs.

So then I skip down and I say okay we can keep counting but let's go ahead and skip down and let's say that we're here at 95, 96, 97, 98, 99, and of course they say what's next is a hundred. And then I ask them how to write a hundred. Of course, they tell me one followed by two zeros. And again I have them turn to a partner, 15 seconds or so, and why do we write it this way? Why do we write one hundred this way?

And so they'll report out and we'll get zero 1's, and zero 10's, and one 100. Which is great. But now we want to generalize this and so what we're going to do is go and we, uh. I write on the board and we talk about the various place values.

So they'll tell me that this is the one's place in base 10 and this is ten and a hundred and a thousand. And we talk about what is the relationship between consecutive or neighboring place values. So, for example, the tens place is 10 times larger than the ones place. The hundreds place is ten times larger than the tens. The thousands place is 10 times larger than a hundred.

So we notice that going to the left each place value is 10 times larger than the place value to its right. We also talked about we could reverse we could go to the to the right and the hundreds place is 1/10th the place value to its left or $1/10^{th}$ of 1000, and likewise 10 is 1/10th of a hundred, one is $1/10^{th}$ of 10.

So there's this relationship between these consecutive place values. Thinking about that if I start here with my tens place then a hundred is 10 times 10, then a thousand is 10 times 100 but I just said 100 was 10 x 10 so 10 x 10 x 10.

So we develop this idea that we can use exponents to describe these place values. And so when I ask them okay well that works really great how do I do it for 1. Somebody will chime up and say well that's just 10 to the zero power.

So this is all well and good this works really well for any base and so we generalize that then. We say okay so if I have any base b, what are my digits? And they'll say 0, 1, 2, up through b – 1. And then, what are the place values? And they'll use what we discovered for base 10. And tell us its b to the 0, b to the first, b square, b cubed.

So they have this framework for dealing with any base. So then I go around and I assigned each group of students, generally it's two, three maybe four students, a different base. Base 2, Base 3, Base 4, Base 5, and so forth. So that they're all working with something slightly different and that they can then present to the class.

So when each group is preparing to present to the class they will they are expected to identify the base as well as the digits that are permitted in that base or that are allowable. They will also identify the various place values. Um, 3 to the 0, let's say for base 3, 3 to the 1, 3 squared, 3 cubed and so forth. Now after they've got this ready I come around and I give them a pile of blocks, of the cubes, and I ask them to tell me how many cubes are in the pile. So they'll count them 2, 4, 6, 8, 10, 12, 13. They'll tell me 13. And I say great that's a base-10 number. Thirteen of them but I want you to tell me how many there are in Base-3. And they'll take their time and discuss it and recognize in Base-3 they need to group by three's. So after a period of time I might see something like this. I'll ask them what is that number? And sometimes I will get 41. And then I'll push back is 4 allowed? Can I write a 4 in this base? And of course, I can't. And then they will eventually recognize that they need to group three of these longs together so that they have 1 flat, this is called a flat, 1 long, and 1 unit. Which means the number they're looking for is 111. So 13 can be written alternatively as 111 in base 3.

And as the groups present the results for their base, we will keep track. They will all have been given the same number of blocks in this case 13. So all of them will have had 13 blocks. One group, the base 3 group, will have expressed this as 111 base 3. Another group will have expressed this quantity of blocks as 1101 base 2. And a different group would have expressed this as 31 in base 4.

So we spend time doing this so that when we move forward and start our study of arithmetic so that students can think carefully about the various operations and the meanings behind the processes that they use in arithmetic. Thank you for joining me.